
Appendix

To help the readers understand how the aggregated functions work, we provide a numerical example here. Imagine a Chinese-English bilingual at the age of 25 and with English as his L2, his answers to many questions of LHQ are listed below.

| Aspects | L1 (Chinese) | | | | L2 (English) | | | |
|------------------------------|--------------|----------|---------|---------|--------------|----------|---------|---------|
| | Listening | Speaking | Reading | Writing | Listening | Speaking | Reading | Writing |
| Age of Acquisition (Q.5) | 1 | 2 | 3 | 5 | 9 | 9 | 9 | 9 |
| Years of Use (Q.5) | 24 | | | | 10 | | | |
| Proficiency (Q.11) | 7 | 7 | 7 | 7 | 6 | 6 | 5 | 4 |
| Hours of Use (Q.14 and Q.15) | 8 | 8 | 4 | 2 | 4 | 4 | 2 | 1 |

Assume that a researcher treats different linguistic aspects (Listening, speaking, reading and writing) equally and assign each aspect a weight of 25% (0.25). The participant's overall proficiency score on Chinese (L1) would be 1.

$$\begin{aligned}
 \textit{Proficiency}_{L1} &= \frac{1}{7} \sum_{j=\{R,L,W,S\}} \omega_j P_{i,j} = \frac{1}{7} (0.25 * 7 + 0.25 * 7 + 0.25 * 7 + 0.25 * 7) \\
 &= 1
 \end{aligned}$$

And his proficiency score on English (L2) would be 0.75

$$\begin{aligned}
 \mathbf{Proficiency}_{L2} &= \frac{1}{7} \sum_{j=\{R,L,W,S\}} \omega_j P_{i,j} = \frac{1}{7} (0.25 * 6 + 0.25 * 6 + 0.25 * 5 + 0.25 * 4) \\
 &= \mathbf{0.75}
 \end{aligned}$$

Please note that a researcher could give different weights to different linguistic aspects to fit his own research purpose. For example, if a researcher only focuses on bilingual participants' listening and speaking aspects of their two languages, he could set the weight to be 0.5 each for listening and speaking, but 0 for the other two. In this case, the participant's overall proficiency score on Chinese (L1) would still be 1.

$$\mathbf{Proficiency}_{L1} = \frac{1}{7} (0.5 * 7 + 0.5 * 7 + 0 * 7 + 0 * 7)$$

And his proficiency score on English (L2) would be 0.86, larger than last calculation when reading and writing were also considered. This makes sense since he is not good at reading and writing in L2.

$$\mathbf{Proficiency}_{L2} = \frac{1}{7} (0.5 * 6 + 0.5 * 6 + 0 * 5 + 0 * 4) = \mathbf{0.86}$$

We can also calculate his immersion scores. For his L1 (Chinese), it would be 0.93

$$\begin{aligned}
 \mathbf{Immersion}_{L1} &= \frac{1}{2} \left[\sum_{j=\{R,L,W,S\}} \omega_j \left(\frac{\mathbf{Age} - \mathbf{AOA}_{i,j}}{\mathbf{Age}} \right) + \left(\frac{\mathbf{YoU}_i}{\mathbf{Age}} \right) \right] \\
 &= \frac{1}{2} \left[0.25 * \left(\frac{25 - 1}{25} \right) + 0.25 * \left(\frac{25 - 2}{25} \right) + 0.25 * \left(\frac{25 - 3}{25} \right) + 0.25 * \left(\frac{25 - 4}{25} \right) + \frac{24}{25} \right] \\
 &= \frac{1}{2} \left[\frac{90}{100} + \frac{24}{25} \right] = \mathbf{0.93}
 \end{aligned}$$

And his proficiency score on English (L2) would be 0.52

$$\begin{aligned}
 \text{Immersion}_{L2} &= \frac{1}{2} \left[\sum_{j=\{R,L,W,S\}} \omega_j \left(\frac{\text{Age} - \text{AOA}_{i,j}}{\text{Age}} \right) + \left(\frac{\text{YoU}_i}{\text{Age}} \right) \right] \\
 &= \frac{1}{2} \left[0.25 * \left(\frac{25 - 9}{25} \right) + 0.25 * \left(\frac{25 - 9}{25} \right) + 0.25 * \left(\frac{25 - 9}{25} \right) + 0.25 * \left(\frac{25 - 9}{25} \right) + \frac{10}{25} \right] \\
 &= \frac{1}{2} \left[\frac{64}{100} + \frac{10}{25} \right] = 0.52
 \end{aligned}$$

When his daily use of the languages is considered, we can also calculate his dominance score for the two languages.

$$\begin{aligned}
 \text{Dominance}_{L1} &= \sum_{j=\{R,L,W,S\}} \omega_j \left[\frac{1}{2} \left(\frac{P_{ij}}{7} \right) + \frac{1}{2} \left(\frac{H_{ij}}{K} \right) \right] = 0.25 \left[\frac{1}{2} \left(\frac{7}{7} \right) + \frac{1}{2} \left(\frac{8}{16} \right) \right] + \\
 &0.25 \left[\frac{1}{2} \left(\frac{7}{7} \right) + \frac{1}{2} \left(\frac{8}{16} \right) \right] + 0.25 \left[\frac{1}{2} \left(\frac{7}{7} \right) + \frac{1}{2} \left(\frac{4}{16} \right) \right] + 0.25 \left[\frac{1}{2} \left(\frac{7}{7} \right) + \frac{1}{2} \left(\frac{2}{16} \right) \right] = 0.671875
 \end{aligned}$$

$$\begin{aligned}
 \text{Dominance}_{L2} &= \sum_{j=\{R,L,W,S\}} \omega_j \left[\frac{1}{2} \left(\frac{P_{ij}}{7} \right) + \frac{1}{2} \left(\frac{H_{ij}}{K} \right) \right] = 0.25 \left[\frac{1}{2} \left(\frac{6}{7} \right) + \frac{1}{2} \left(\frac{4}{16} \right) \right] + \\
 &0.25 \left[\frac{1}{2} \left(\frac{6}{7} \right) + \frac{1}{2} \left(\frac{4}{16} \right) \right] + 0.25 \left[\frac{1}{2} \left(\frac{5}{7} \right) + \frac{1}{2} \left(\frac{2}{16} \right) \right] + 0.25 \left[\frac{1}{2} \left(\frac{4}{7} \right) + \frac{1}{2} \left(\frac{1}{16} \right) \right] = 0.4609
 \end{aligned}$$

Therefore the Ratio of Dominance for L1 will be 1 and the Ratio of Dominance for L2 will be

$$\text{Ratio}_{\text{Dominance}_{L2}} = \frac{\text{Dominance}_{L2}}{\text{Dominance}_{L1}} = \frac{0.4609}{0.671875} = 0.686$$